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**SCHUMPETER, KEYNES AND KALDOR'S  
COMPLEMENTARY VIEWS FOR UNDERSTANDING  
ECONOMIC GROWTH**

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*Abstract*

For Schumpeter, economic development has the characteristics of a process of evolution whose central figure is the entrepreneur. For Keynes, entrepreneurs take decisions concerning the volume of output and employment according to an expectation of demand: the “effective demand”. For Kaldor, economic growth is the resultant of a chain-reaction between increases in supply and increases in demand.

In order to show the interest of this vision, I represent this growth process by an entrepreneurial growth model based on the principle of effective demand. The aggregate supply function makes use of Keynes' and Schumpeter's complementary views of the entrepreneur's behavior.

The growth process is a process of continuing disequilibrium, but in the long term steady states can be found. They have unexpected theoretical properties: the output growth rate is a linear function of the employment growth rate and of the investment rate (or the saving rate), while the profit share in the income is exactly 1/3.

The theoretical lessons turn out to be consistent with the realities of the American economy from 1960 to 2000. Those results back up the Schumpeter, Keynes and Kaldor's complementary views.

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## **Introduction**

For Schumpeter (1911, 1942), entrepreneurs establish new productive combinations in order to “produce more”, through capacity investments, or to “produce differently”, through process investments. For Keynes (1936), entrepreneurs make decisions concerning investment and employment according to the effective demand, the marginal efficiency of capital and the marginal propensity to consume.

After the publication of *General Theory* by Keynes, Kaldor carried out a series of works studying the economic growth process (1956, 1961, 1972), more precisely the link between this process and the principle of effective demand, capital accumulation, increasing returns and technical progress. “Given that factor, the process of economic development can be looked upon as the resultant of a continued process of interaction –one could almost say, of a chain-reaction- between demand increases which have been induced by increases in supply, and increases in supply which have been evoked by increases in demand” Kaldor (1972, p. 1246) concluded. That view has never been represented into a growth model.

I represent Kaldor’s view of economic growth with an entrepreneurial growth model (Ebner, 2000) based on the principle of effective demand. The aggregate demand function is a classical one, while the aggregate supply function makes use of Keynes’ and Schumpeter’s complementary views of the entrepreneur’s behavior. Obviously, the equilibrium of effective demand is not reached – allowing for exceptions - and for the following period of time entrepreneurs expect a new equilibrium of effective demand. I shall show that, in the long term, growth process allows steady states when entrepreneurs’ anticipations have been fulfilled in reality and when growth is balanced. The theoretical lessons to be drawn are unexpected: the output growth rate is a linear function of the employment growth rate and of the investment rate (or the saving rate), while the profit share in the income is exactly 1/3.

The theoretical lessons turn out to be consistent with the numerous stylized facts listed by Kaldor (1961), Barro and Sala-i-Martin (1995) as well as with the realities of the American economy from 1960 to 2000.

In the first section of the paper, I will explain the points of view -complementary in many aspects- of Schumpeter and Keynes on entrepreneurs’ behavior and the view of Kaldor on the economic growth process. In the second section, the growth process described by Kaldor will be represented by an entrepreneurial growth model based on the principle of effective demand. In the third section, I will identify the steady states of the growth process. The major insights will be outlined in the fourth section. In the fifth section the major theoretical insights will be compared with the stylized facts as well as with the realities of the American economy from 1960 to 2000.

### **1. The Schumpeterian-Keynesian foundations and the Kaldor’s view**

For Schumpeter economic development has the characteristics of a process of evolution whose central figure is the entrepreneur (Nelson and Winter, 1982; Nelson, 2005, 2007; Hanusch and Pyka, 2007). As Nelson and Winter (1982) have emphasized, the process of evolution is one of continuing disequilibrium,<sup>2</sup> given the decisions taken by entrepreneurs.

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<sup>2</sup> P. 276 :“Although these models have yielded some illuminating insights, they ignore essential aspects of Schumpeterian competition - the fact that there are winners and losers and that the process is one of continuing disequilibrium.”

This process is led by entrepreneurs who have identified opportunities for creating wealth via implementation of innovations.<sup>3</sup>

Schumpeter makes the distinction between “to produce more” and “to produce differently”.<sup>4</sup> “To produce more” mainly covers additional production of existing products or production of new products; dissemination of product innovations is part of this. “To produce differently” mainly covers the transformation of production process with, for example, lower product costs; dissemination of process innovations is part of this. This distinction between “to produce more” and “to produce differently” is a most fundamental one, for the entrepreneur's decisions must take it into account. According to the types of investment, innovation can contribute either to create employments or to shed some of them (Goodwin, 1991; Lorenzi and Bourles, 1995; Pianta, 2006;<sup>5</sup> Crespi and Pianta, 2008), which reflects the creative destruction at work.

For Schumpeter (1942, p. 97), the entrepreneur's aim is competitiveness, as the competitive goods market obliges producers to opt for the lowest total cost per unit of production: “Everyone agrees that private and socialist managements will introduce improvements if, with the new method of production, the total cost per unit of product is expected to be smaller than the prime cost per unit of product with the method actually in use”. In addition, profit is in essence “the result of carrying out new combinations” (Schumpeter, 1911, p. 136). In other words, the entrepreneur takes the production, investment and employment decisions so as to obtain the lowest total cost per unit of product, while drawing on available innovations; profit maximization becomes a more long-term goal.

In his *General Theory* (1936),<sup>6</sup> Keynes considers that entrepreneurs take decisions concerning the volume of output and employment according to an expectation<sup>7</sup> of demand: the “effective demand”. This effective demand<sup>8</sup> is given by the point of the intersection between the aggregate supply function  $Z = \varphi(L)$  and the aggregate demand function  $D = f(L)$ ; at this point (the equilibrium of effective demand), the entrepreneurs' expectation of profits will be maximized (Davidson, 2001).

The employment determinants are mainly “the propensity to consume” and “the inducement to invest”. This latter is determined by the comparison between the marginal efficiency of capital, in other words the rate of return to capital which measures the expected yield from an investment, and the real interest rate. Thus the entrepreneur will only invest if the marginal efficiency of capital is higher than the real interest rate.

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<sup>3</sup> Let's quote Nelson (2007, p. 37): “economic growth needs to be understood as a process driven by the coevolution of physical and social technologies.”

<sup>4</sup> Schumpeter, 1926 (second edition), p. 121. It should be remembered that for Schumpeter (1911, p. 65), “To produce means to combine materials and forces within own reach...To produce other things, on the same things by a different method, means to combine these materials and forces differently.”

<sup>5</sup> Pianta (2006) has highlighted the stylized fact that product innovation helps create jobs while process innovation destroys them: “The evidence shows that it is essential to discriminate between product innovation (novel or imitation) that has a generally positive employment impact and process innovation (adoption or use of new technologies) usually with negative effects” (p. 590).

<sup>6</sup> Chapter 5: “Expectation as determining Output and Employment” (p. 46).

<sup>7</sup> For Keynes (1936, p. 46), the entrepreneur “has no choice but to be guided by these expectations, if he is to produce at all by processes which occupy time.”

<sup>8</sup> “The value of D at the point of the aggregate demand function, where it is intersected by the aggregate supply function, will be called the effective demand.” (Keynes, 1936, p. 25).

And so for Keynes (1936)<sup>9</sup>, short-term decisions must take account of long-term expectations regarding investment. Marginal efficiency of capital is thus a long-term expectation which depends on the state of confidence of the entrepreneurs and cannot be replaced by the current rate of return to capital.

With regard to the search for maximum profit (*ex ante*) undertaken by the entrepreneur, Keynes stresses the constraint represented by the “amount of employment to offer”.<sup>10</sup> Keynes (1936, p. 141) also highlights the risk of long-term competition the entrepreneur takes: “The output from equipment produced to-day will have to compete, in the course of its life, with the output from equipment produced subsequently, perhaps at a lower labor cost, perhaps by an improved technique... Moreover, the entrepreneur's profit (in terms of money) from equipment, old or new, will be reduced, if all output comes to be produced more cheaply.”. The risk is all the greater in that the prospective yield is substantial and will bring other entrepreneurs onto the market.

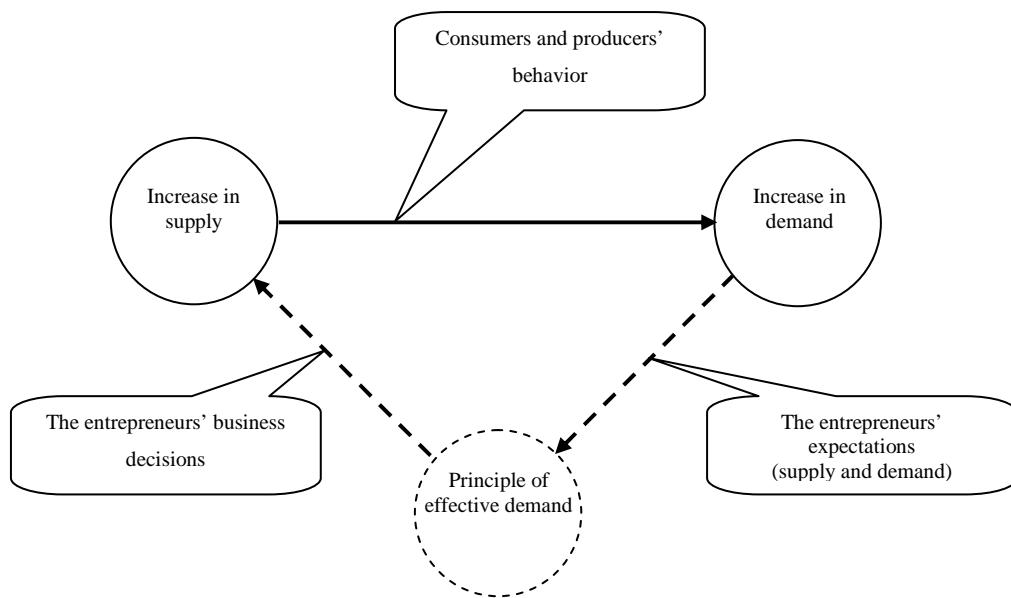


Figure 1 - The growth process as a chain-reaction

“Workers will not seek a much greater money-wage when employment improves.” This conclusion by Keynes (1936, p. 253) has been widely confirmed by empirical research. In his investigation of 27 industrial sectors within the American economy between 1923 and 1950, Salter (1960, 1966) noted the lack of correlation between labor productivity gains<sup>11</sup> and employment growth. Other economists have also highlighted this fact in the United States in similar terms: there is no correlation between labor productivity and employment (Hansen and Wright, 1992).

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<sup>9</sup> “Nevertheless, we must not forget that in the case of durable goods, the producer's short-term expectations are based on the current long-term expectations of the investor...Thus the factor of current long-term expectations cannot be even approximately eliminated or replaced by realized results” (p. 51).

<sup>10</sup> See for example Keynes (1936, p. 23-24): “The entrepreneur's profit thus defined is, as it should be, the quantity which he endeavors to maximize when he is deciding what amount of employment to offer.”

<sup>11</sup> The profit share in the income being a constant on the long term, the wage growth rate is equal to the labor productivity growth rate.

And so, while Schumpeter stresses the aim of “short-term” competitiveness, Keynes foregrounds the aim of “long-term” competitiveness. In fact these two different points of view characterize two indissociable aspects of competitiveness.

Thus, Keynes and Schumpeter's views are complementary<sup>12</sup> since they concern both the process of evolution resulting from entrepreneurs who take investment and innovation risks, and the process of entrepreneurs' decisions about production, investment and employment.

For Kaldor (1972), the growth process as a chain-reaction between increases in supply and increases in demand is understood when the principle of effective demand and increasing returns are taken into account. I represent this growth process by the figure 1.

## 2. An entrepreneurial growth model based on the principle of effective demand

The new entrepreneurial growth model (figure 2) that we are presenting is based on the principle of effective demand for the short term period  $[t, t+dt]$ . The aggregate supply function, represented by  $\dot{Z}^a = \phi(\dot{L}^a)$ , is the additional (expected) volume of output resulting from the employment increase  $\dot{L}^a$ .<sup>13</sup> The aggregate demand function, represented by  $\dot{D}^a = f(\dot{L}^a)$ , is the additional proceeds which entrepreneurs expect to receive from employment increase  $\dot{L}^a$ . Let  $\dot{D}^{a,e}$  be the additional effective demand, the additional demand given by the point of intersection (for employment increase  $\dot{L}^{a,e}$ ) between the aggregate supply function and the aggregate demand function.

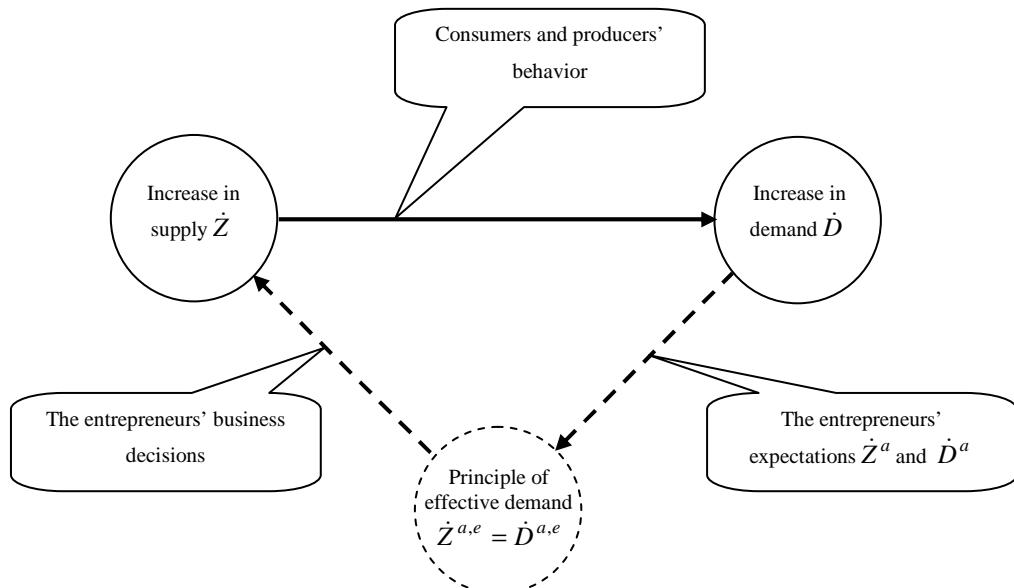


Figure 2 - The growth process as a chain-reaction

Classically, the aggregate demand function that we are looking at is the same kind as Keynes'. However, the aggregate supply function is based on the complementary views of

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<sup>12</sup> Many publications dealt with the possible synthesis between Keynes' and Schumpeter's view (for example, Minsky, 1986; Goodwin, 1991, 1993; Davidson, 2002; Bertocco, 2007).

<sup>13</sup>  $a$  indicates the anticipated character of the variable (or ex ante).

Schumpeter and Keynes. Entrepreneurs aim at creating new productive combinations by taking innovation opportunities. They try to “produce more” or “produce differently”.

Entrepreneurs implement capital and labor factors to meet an anticipated additional demand.<sup>14</sup> They expect one part of the net investment<sup>15</sup> to be a capacity investment associated with additional supply (“to produce more”) while the other part is to be a process investment (“to produce differently”) associated with stagnant demand. The first type of investment will be called here “capacity investment”, with capital and labor coming together “to produce more” (with increasing returns). The second type will be called “process investment”, with capital taking the place of labor. While employment creation is associated with capacity investment, employment destruction is associated with process investment, thus reflecting the process of creative destruction at work.<sup>16</sup>

Investment and employment decisions are made according to the marginal efficiency of the capital that entrepreneurs consider for their investment project. Entrepreneurs aim at minimizing the total cost (per unit of additional supply) linked to the capacity investment they plan, taking into account the costs resulting from the workforce and the capacity investment. The minimization of the total cost can only take place under a constraint related to the amount of employment to be created. This constraint reflects the risk of being confronted to new competitors, this risk being all the more important since the marginal efficiency of the capital is substantial.

According to the principle of effective demand, the new equilibrium expected by entrepreneurs is the intersection of the aggregate supply function and the aggregate demand function. Now, I shall determine the aggregate supply function, then the aggregate demand function and the equilibrium of effective demand. At the initial time  $t$ , I assume that the supply  $Z$ , the demand  $D$ , and the production  $Y$  are in equilibrium ( $Z = D = Y$ ).

## **2.1. The aggregate supply function**

Taking into account the relationship between supply and investment, the effect of creative destruction on employment, and the necessary competitiveness of the new productive combinations, the aggregate supply function  $\dot{Z}^a$  is defined.

### The supply-investment relationship

Entrepreneurs anticipate an additional supply  $\dot{Z}^a$  which they intend to meet by a net investment  $I_n^a$ , which comprises a capacity investment  $x^a I_n^a (0 \leq x^a \leq 1)$  and a process investment  $(1 - x^a) I_n^a$ ;  $x^a$  is the share of the capacity investment engaged in the total volume of net investment. Entrepreneurs use the technology  $\mathcal{A}$  characterized by a supply function<sup>17</sup> of the type  $\dot{Z} = A \dot{K}$ ,  $A$  being the marginal rate of productivity of the capital associated with the capacity investment;<sup>18</sup>  $A$  will henceforth be termed “productivity of the capacity investment”.

<sup>14</sup> As Kaldor (1972, p. 1240), we assume that the markets are “instrument for transmitting impulses to economic change”.

<sup>15</sup> The net investment is the (gross) investment minus the replacement investment.

<sup>16</sup> Thus, capital and labor inputs are in part substitutable and in part complementary with increasing returns to scale.

<sup>17</sup> This function is taken, via differentiation in relation to time, from the form ( $Y=AK$ ) proposed by Harrod (1939) and Domar (1947) but also by Nelson and Winter (1982) to represent Schumpeter's analysis.

<sup>18</sup> In accordance with the Nelson-Phelps (1966) approach, it is acknowledged that human capital is not a production factor; thus the variable of the capital  $K$  will not include human capital.

The productivity of the capacity investment is then assumed to remain constant. The additional supply will be met as follows:<sup>19</sup>

$$\dot{Z}^a = Ax^a I_n^a \quad \text{with} \quad I_n^a = \dot{K}^a \quad 0 \leq x^a \leq 1 \quad \Rightarrow \quad \frac{\dot{Z}^a}{Z} = Ax^a \frac{I_n^a}{Z} \quad (1)$$

$x^a$  will be called anticipated “growth multiplier”, with any increase in  $x^a$  leading to faster growth in supply.

### The creative destruction

Entrepreneurs plan to create jobs according to the additional supply, supply-employment elasticity being variable:

$$\frac{\dot{Z}^a}{Z} = e_c^a \frac{L_c^a}{L} \quad e_c^a > 1 \quad (2)$$

where  $L_c^a$  is employment creation associated with capacity investment  $x^a I_n^a$ . It is accepted that the jobs created are more productive, given the innovation brought by the investment and the increasing returns; whence an elasticity higher than 1. Thus the employment creation anticipated by the entrepreneur is:<sup>20</sup>

$$L_c^a = \frac{A}{e_c^a} \frac{L}{Y} x^a I_n^a = \varepsilon_c^a \frac{L}{Y} x^a I_n^a \quad \text{with} \quad \varepsilon_c^a = \frac{A}{e_c^a} \quad 0 < \varepsilon_c^a < A \quad (3)$$

The coefficient  $\varepsilon_c^a$ , called anticipated “employment creation coefficient”, is obviously variable, given the variable character of elasticity.<sup>21</sup> In the same way the entrepreneurs plan to cut employment according the “supply deficit”  $A(1 - x^a)I_n^a$ , elasticity also being assumed as variable. Thus the employment destruction  $L_l^a$  anticipated by the entrepreneurs is:

$$L_l^a = \varepsilon_l^a \frac{L}{Y} (1 - x^a) I_n^a \quad \varepsilon_l^a \geq 0 \quad (4)$$

The coefficient  $\varepsilon_l^a$ , called anticipated “employment destruction coefficient”, is also variable. I assume now that there is a link between the choices of employment creation and employment destruction, and clarify it via two considerations. When the entrepreneurs choose highly creative combinations of jobs, the combinations chosen for capital-labor substitution destroy fewer jobs; this reflects confidence in additional supply in terms of the firm's products as a whole. Furthermore there are limits to the creation and destruction of employment, these limits depending on the technology used. These choices of productive combinations are reflected in the following relationships:

$$\varepsilon_c^a + \varepsilon_l^a = \varepsilon_c^{mx} \quad 0 < \varepsilon_c^a \leq \varepsilon_c^{mx} \quad 0 \leq \varepsilon_l^a < \varepsilon_c^{mx} \quad \varepsilon_c^{mx} < A \quad (5)$$

where  $\varepsilon_c^{mx}$  is the maximum coefficient of employment creation,<sup>22</sup> a characteristic of technology  $\mathcal{A}$ . This latter parameter reflects the organizational limit of the creation or destruction of employment, given the forms of organization set up by the entrepreneurs in the context of the use of technology  $\mathcal{A}$ . Thus, the anticipated increase in employment is:

<sup>19</sup> This function is also consistent with the acceleration principle of investment (Harrod, 1939).

<sup>20</sup> At the beginning of the short-term period:  $Z=D=Y$ .

<sup>21</sup> Keynes (1936, p. 286) clearly considers demand-employment elasticity as highly variable: “If, for example, the increased demand is largely directed towards products which have a high elasticity of employment, the aggregate increase in employment will be greater than if it is largely directed towards products which have a low elasticity of employment.”

<sup>22</sup> It is also the limit of the maximum coefficient of employment destruction.

$$\dot{L}^a = L_c^a - L_l^a = (\varepsilon_c^{mx} x^a + \varepsilon_c^a - \varepsilon_c^{mx}) \frac{L}{Y} I_n^a \quad 0 < \varepsilon_c^a \leq \varepsilon_c^{mx} \quad \varepsilon_c^{mx} < A \quad (6)$$

The increase in employment depends on the values anticipated for the variables of the volume of net investment, of the growth multiplier and of the employment creation coefficient.

#### The competitive productive combination

In order to meet an additional anticipated supply  $\dot{Z}^a$ , in the context of a marginal efficiency of capital  $e_K$ , entrepreneurs can implement various productive combinations involving the growth multiplier  $x^a$ , the employment creation coefficient  $\varepsilon_c^a$  and the volume of investment  $I_n^a$ . They will then decide on the combination that fits with the aim of the lowest cost and the aim of expected profit. The appendix 1 shows that the conditions are the following:

$$x^a = \frac{e_K}{cA} \quad \varepsilon_c^a = \frac{e_K}{1-c} \quad \varepsilon_c^{mx} = \frac{cA}{1-c} \quad 0 < x^a \leq 1 \quad c < \frac{1}{2} \quad (7)$$

$c$  being the profit share in the income. Hence:

$$\dot{Z}^a = \frac{e_K}{c} I_n^a \quad \dot{L}^a = \frac{(2e_K - cA)}{(1-c)} \frac{L}{Y} I_n^a \quad (8)$$

Thus the aggregate supply function is defined as following:

$$e_K \neq \frac{cA}{2} \text{ and } e_K \leq cA \quad \dot{Z}^a = \varphi(\dot{L}^a) = \frac{(1-c)e_K}{c(2e_K - cA)} \frac{Y}{L} \dot{L}^a \quad \text{with} \quad \dot{L}^a = \frac{(2e_K - cA)}{(1-c)} \frac{L}{Y} I_n^a \quad (9)$$

$$e_K = \frac{cA}{2} \quad \dot{Z}^a = \frac{A}{2} I_n^a \quad \dot{L}^a = 0 \quad (10)$$

The aggregate supply function (figure 3) is a straight line whose slope is a decreasing function of the marginal efficiency of capital.<sup>23</sup>

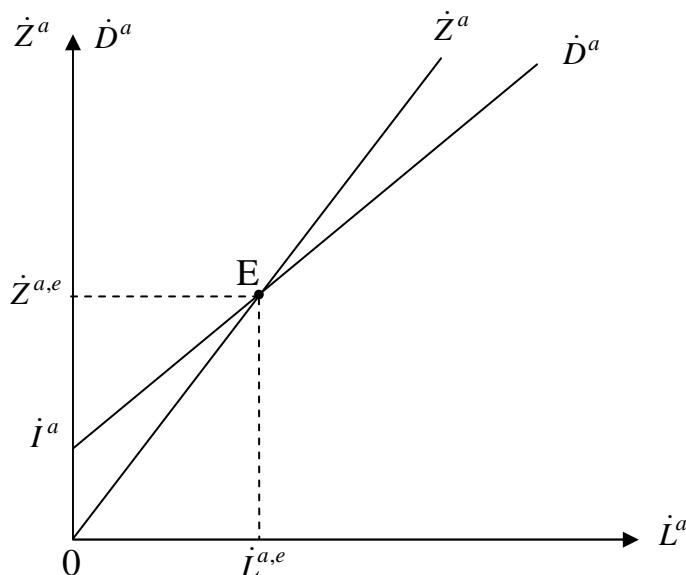


Figure 3 - Aggregate supply function and aggregate demand function

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<sup>23</sup> The marginal efficiency of capital is less than  $cA$  (growth multiplier less than 1).

## 2.2. The aggregate demand function

The aggregate demand function is the additional volume that the community (producers and consumers) is expected to spend on consumption and to devote to investment, taking into account the anticipated increase employment. We suppose that the (anticipated) marginal propensity to consume is  $p_C^a$ . Hence:

$$\dot{D}^a = p_C^a \dot{Z}^a + \dot{I}^a \quad 0 < p_C^a < 1 \quad (11)$$

The anticipated additional demand is a function of the anticipated marginal propensity to consume, of the aggregate supply function, and of the anticipated increase of the investment. Hence:

$$\dot{D}^a = f(\dot{L}^a) = p_C^a \frac{(1-c)e_K}{c(2e_K - cA)} \frac{Y}{L} \dot{L}^a + \dot{I}^a \quad (12)$$

The aggregate demand function (figure 3) is also a straight line that intersects the aggregate supply function. We assume that the relation between net investment and investment is simply:  $I_n^a = (1 - \delta)I^a$  where  $\delta$  is the replacement investment rate, a constant in time. Hence:

$$\dot{D}^a = f(\dot{L}^a) = p_C^a \frac{(1-c)e_K}{c(2e_K - cA)} \frac{Y}{L} \dot{L}^a + \frac{\dot{I}_n^a}{(1-\delta)} \quad (13)$$

## 2.3. The equilibrium of effective demand

In a general case, the equilibrium is determined by the intersection of the two functions (figure 3):

$$\dot{L}^{a,e} = \frac{c(2e_K - cA)}{(1-p_C^a)(1-c)(1-\delta)e_K} \frac{Y}{L} \dot{I}_n^a \quad \dot{Z}^{a,e} = \dot{D}^{a,e} = \frac{\dot{I}_n^a}{(1-p_C^a)(1-\delta)} = \frac{e_K}{c} I_n^a \quad (14)$$

The employment increase depends on the marginal propensity to consume, on the marginal efficiency of capital, and on the net investment increase. Here, we find a result that is consistent with Keynes' *General Theory* and we recognize, in the latest equation, the expression of investment multiplier  $1/(1-p_C^a)$ .

Thus, after having defined the marginal efficiency of capital  $e_K$ , the marginal propensity to consume  $p_C^a$  and the volume of investment  $I_n^a$ , entrepreneurs are able to determine the additional effective demand  $\dot{D}^{a,e}$ , the employment increase  $\dot{L}^{a,e}$ , the growth multiplier  $x^a$  and the investment increase  $\dot{I}_n^a$ .

$$\dot{D}^{a,e} = \dot{Z}^{a,e} = \frac{e_K}{c} I_n^a \quad \dot{L}^{a,e} = \frac{(2e_K - cA)}{(1-c)} \frac{L}{Y} I_n^a \quad x^a = \frac{e_K}{cA} \quad \dot{I}_n^a = (1-p_C^a)(1-\delta) \frac{e_K}{c} I_n^a \quad (15)$$

The entrepreneur's choices can be readily interpreted: when he anticipates a rise in marginal efficiency, he reduces the volume of investment, while anticipating a higher growth multiplier (a higher proportion of capacity investment) and greater employment growth so as to keep on coping with the same additional effective demand.

It is important to note here that the growth rates for demand and employment which are anticipated for the short term are linked by a linear relationship independent of the marginal efficiency of capital, in other words, independent of the view entrepreneurs formulate for the long term:

$$\frac{\dot{D}^{a,e}}{D} = \frac{\dot{Z}^{a,e}}{Z} = \frac{1-c}{2c} \frac{\dot{L}^{a,e}}{L} + \frac{A}{2} \frac{I_n^a}{Y} \quad (16)$$

### 3. The stationary states of an economy

The growth process is made of a succession of increases of supply and demand, induced by a succession of equilibriums of the effective demand. I shall show that, in the long term, stationary states do exist and that these are steady states<sup>24</sup> (Aghion and Howitt, 1998) within which growth rates for output and employment are constant over time. To identify stationary states I make the classical assumption that the entrepreneur's anticipations have been fulfilled in reality, that the marginal propensity to consume is a constant over time,<sup>25</sup> and that growth is balanced, notably in the sense of the work of Harrod (1939, 1948) and Domar (1947). On the long term, I suppose that there is no correlation between the labor productivity growth rate and the employment growth rate.

#### 3.1. The determination of stationary states

##### Anticipations and realities in the long term

The anticipated values of the fundamentals encounter reality:

$$\dot{D}^{a,e} = \dot{Z}^{a,e} = \dot{D} = \dot{Z} = \dot{Y} \quad \dot{L}^{a,e} = \dot{L} \quad I_n^a = I_n \quad x^a = x \quad \dot{I}_n^a = \dot{I}_n \quad p_{\dot{C}}^a = p_{\dot{C}} \quad e_K = q = \frac{\dot{P}}{\dot{K}} \quad (17)$$

where  $P$  is the profit (equal to  $cY$ ) and  $q$  is the rate of return to capital.<sup>26</sup> In the long term, for an additional output  $\dot{Y}$ , entrepreneurs decide to invest  $I_n$  and to increase employment by  $\dot{L}$  and investment by  $\dot{I}_n$  according to the following formulae:<sup>27</sup>

$$\dot{Y} = \frac{q}{c} I_n \quad \dot{L} = \frac{(2q - cA)}{(1-c)} \frac{L}{Y} I_n \quad x = \frac{q}{cA} \quad \dot{I}_n = (1 - p_{\dot{C}})(1 - \delta) \frac{q}{c} I_n \quad (18)$$

In addition, the equality between supply and demand (and between the increases) implies the following formulae:

$$\frac{I}{Y} = \frac{E}{Y} = i = s = 1 - p_c \quad \frac{\dot{I}}{\dot{Y}} = \frac{\dot{E}}{\dot{Y}} = 1 - p_{\dot{C}} \quad (19)$$

where  $i, s, p_c$  are the (gross) investment rate, the (gross) saving rate and the average propensity to consume. We assume that the marginal propensity to consume is a constant equal to the mean propensity to consume. Hence:

$$p_{\dot{C}} = p_c = \text{constant} \Rightarrow \frac{I}{Y} = \frac{E}{Y} = \frac{\dot{I}}{\dot{Y}} = \frac{\dot{E}}{\dot{Y}} = i = s = 1 - p_c = \text{constant} \quad (20)$$

Thus, the net investment rate and the net saving rate are constant on the long term.

##### Balanced growth in the Harrod and Domar sense

We assume that the output growth rate is equal to the capital growth rate ("warranted" growth rate); in other words the mean productivity of the capital is a constant for the long term:

$$\frac{\dot{Y}}{Y} = \frac{\dot{K}}{K} \Leftrightarrow \frac{Y}{K} = \frac{\dot{Y}}{\dot{K}} \Leftrightarrow y_K = \frac{Y}{K} = \text{constant} \Rightarrow \frac{\dot{Y}}{Y} = y_K i_n = \text{constant} \quad (21)$$

Given the relations (18):

$$\frac{\dot{Y}}{Y} = y_K i_n = \frac{q}{c} i_n = Axi_n = \text{constant} \quad (22)$$

<sup>24</sup> The research of steady states turn out to be a very good analytic tool (Palley, 1996).

<sup>25</sup> In the same way Keynes (1936) considers that the propensity to consume is a stable function.

<sup>26</sup> Here, the profit share in the income is not assumed constant.

<sup>27</sup> From relations (15).

There are three consequences. The first consequence is the constancy of the growth multiplier:

$$x = \frac{y_K}{A} = \text{constant} \quad (23)$$

The second consequence is the constancy of the profit share in the income:

$$q = cy_K \quad \text{and} \quad q = \frac{\dot{c}Y + c\dot{Y}}{\dot{K}} \Rightarrow c = \text{constant} \quad (24)$$

The third consequence is the constancy of the (mean) return on capital  $z$  (equal to the rate of return to capital):

$$z = c \frac{Y}{K} = cy_K = q = \text{constant} \quad (25)$$

Thus stationary states are characterized by the following equations:

$$\begin{aligned} \frac{\dot{Y}}{Y} &= Ax_i_n & \frac{\dot{L}}{L} &= \frac{cA}{1-c}(2x-1)i_n & y_K &= Ax & z &= cAx \\ 0 < x \leq 1 & & c < \frac{1}{2} & & x &= \text{constant} & i_n &= s_n = \text{constant} & c &= \text{constant} \end{aligned} \quad (26)$$

We note that the relationship between output growth rate and employment growth rate is independent of the growth multiplier:

$$\frac{\dot{Y}}{Y} = \frac{1-c}{2c} \frac{\dot{L}}{L} + \frac{A}{2} i_n \quad (27)$$

### 3.2. The independence between wage growth and employment growth

In his *General Theory* Keynes stresses that wage growth and employment growth are independent of each other. In other words the labor market induces a standard wage increase that is imposed on all firms, whatever the employment growth rate. The wage growth rate must thus be independent of the employment growth rate. But given the invariability of the profit share in the income, the wage growth rate is equal to the rate of growth of labor productivity. Hence:

$$\frac{\dot{\omega}}{\omega} = \frac{\dot{Y}}{Y} - \frac{\dot{L}}{L} = \left[ \frac{1-c}{2c} - 1 \right] \frac{\dot{L}}{L} + \frac{A}{2} i_n \quad \frac{1-c}{2c} - 1 = 0 \Leftrightarrow c = \frac{1}{3} \quad (28)$$

An immediate, striking consequence lies in the profit share in the income, which is invariably equal to 1/3. As a consequence, in the long term, the entrepreneur must introduce productive combinations<sup>28</sup> in which the profit share in the income is equal to 1/3.

### 3.3. The existence of steady states

For the stationary states, the growth rates for output and employment remain constant over time. These stationary states thus have the property of steady states. Table 1 recapitulates the expression of the main fundamentals in steady states: they are expressed simply, according to the productivity of the capacity investment, the net investment rate (or saving rate) and the growth multiplier.

The productivity of the capacity investment and the investment rate are exogenous data. The former reflects the speed of technological progress allowed by the technologies used and the accompanying institutions. Thus it does not reflect the level of technological progress; a technologically backward economy can be characterized by productivity of the capacity

<sup>28</sup> The maximum coefficient of employment creation is equal to half of productivity of the capacity investment.

investment higher than that of an advanced economy. The investment rate depends on the monetary conditions (not taken into account in the growth model).

Fundamentals	Equations
Output growth rate	$\frac{\dot{Y}}{Y} = \frac{\dot{K}}{K} = Ax i_n$ $0 < x \leq 1 \quad x = \text{constant}$ $i_n = s_n = \text{constant}$
Employment growth rate	$\frac{\dot{L}}{L} = \frac{A(2x-1)}{2} i_n$
Profit share in income	$c = \frac{1}{3}$
Capital productivity	$y_K = \frac{Y}{K} = Ax$
Return on capital	$z = \frac{A}{3} x$
$A$ : productivity of the capacity investment	$x$ : growth multiplier
$i_n$ : net investment rate	$s_n$ : net saving rate

Table 1 - Expression of fundamentals in steady states

## 4. Major insights

### 4.1. Significant results in the long term

The first significant result lies in the fundamental output-employment relationship as verified by steady states:

$$\frac{\dot{Y}}{Y} = \frac{\dot{L}}{L} + \frac{A}{2} i_n \quad \text{with} \quad i_n = s_n = \text{constant} \quad (29)$$

The labor productivity growth rate is simply equal to the half-product of the net investment rate (or net saving rate) and the productivity of the capacity investment.

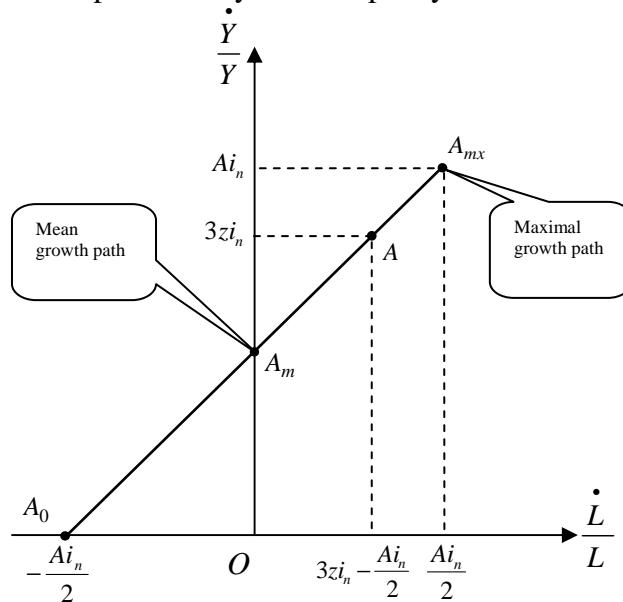


Figure 4 - The steady states

The set of stationary states is represented by  $A_0A_{mx}$  in Figure 4, the point  $A_0$  being excluded. The slope of this segment is simply equal to the unit. The steady state associated with the return on capital  $z$  is the point  $A$ .  $A_{mx}$  represents the maximal growth path (for the long term); the output and employment growth rates are thus maximal, all the new productive combinations being engaged in increasing returns.  $A_m$  represents the mean growth path, the growth multiplier being equal to 0.5.

As a rule, the larger the share of investment committed to complementarity (production factors), the stronger the growth - and the return on capital. Put otherwise, the more entrepreneurs succeed in becoming involved in increasing returns, the higher the growth and the greater the return on capital.

The second significant result is to be found in the distribution of income: 2/3 for labor income, 1/3 for capital income. This means that whatever the technology  $\mathcal{H}$ , the output growth rate, the employment growth rate, and the investment rate (or saving rate), the profit share in the income is a remarkable constant in the steady states.

The third significant result lies in the quest by entrepreneurs for maximum profit, an incentive for the entrepreneurs to push up the growth multiplier (or the capital productivity) that is to say to further fuel their investments with capacity investments:

$$\text{Max} \left\{ \dot{P} = \frac{1}{3} A x i_n Y \right\} \quad \text{with} \quad i_n = \text{constant} \quad \Rightarrow \quad x \uparrow \quad (30)$$

If this strategy is born out by subsequent events (ex post), the entrepreneur will continue with it. The trajectory of the economy will then be a succession of steady states interrupted by periods of disequilibrium, with growth potential rising as a long-term trend.

#### **4.2. From disequilibrium to steady states**

The decisions taken by entrepreneurs instigate an authentic process of economic evolution. By its very nature this process is unstable, as reality continuously resists all prediction. Nonetheless, the process does possess stationary states in the long term when, classically, it is assumed that anticipations coincide with reality and growth is balanced. How then are we to interpret ongoing disequilibrium and the existence of a set of stationary states?

As a rule growth paths appear to be in disequilibrium, for example when competitiveness is not ensured because entrepreneurs have not attained the lowest unit production cost. One reason for these inappropriate choices classically results, as many economists have shown,<sup>29</sup> from dependence on the technological trajectory. Taking competition into account, these entrepreneurs are then obliged in the long run to come up with adaptive anticipations: in other words, to adopt more competitive productive combinations or to fail (Nelson, 2005). The return to a competitive situation represented by steady states is thus obligatory for these entrepreneurs; however, in the periods that follow, other entrepreneurs are in danger of being non-competitive.

In continuous disequilibrium, the economic trajectories are thus going to endlessly get nearer to and further away from steady states, i.e. the segment  $A_0A_{mx}$ . Thus the steady states are seen fairly much as "attractors" (Nelson, 2005;<sup>30</sup> Villemeur, 2008). Considering that the behavior

<sup>29</sup> See David (2000) for example.

<sup>30</sup> "In their analysis of certain economic phenomena, for example technological progress, many economists recognize that frequent or continuing shocks, generated internally or externally, may make it hazardous to

of entrepreneurs, producers, consumers and markets is perfect, the attractor of steady states symbolizes an ideal chain-reaction.

When the net investment rate is constant, the attractor  $A_0 A_{mx}$  has a double function on the long term: to represent the mean trajectory of the economy, and also to attract economic trajectories<sup>31</sup> (for example annual trajectories). Thus, the means of the fundamentals (output growth rate and employment growth rate) should belong to the attractor, while the latter should be identical to the linear regression established on the long term.

## 5. Comparison with empirical reality

### 5.1. Comparison with stylized facts

Via analysis of the fundamentals of the main economies of the 19th and 20th centuries, Kaldor (1961) has identified six stylized facts characterizing long-term economic growth. For Barro and Sala-i-Martin (1995) these facts are confirmed by the long-term data relative to today's developed countries. It is easy to verify that the theoretical lessons of the growth model are potentially consistent with the stylized facts listed by Kaldor and Barro and Sala-i-Martin.

An income distribution of 1/3 for capital and 2/3 for labor has often been put forward, as numerous historical references testify. In Cobb-Douglas's first growth model (1928), the profit share in the income is a constant parameter evaluated at 30%. Worthy of mention is the evaluation for the United States in the years 1909-1949, with an average of 34%<sup>32</sup> (Solow, 1957). An average share of 34% is also found for a set of economies around the year 1990 (Gollin, 2002).<sup>33</sup>

The income split between profit and wage has been well measured since the 1960s in respect of the main developed economies (European Commission, 2002). The largest developed economy -the United States- has always had a profit share<sup>34</sup> of 30-33%, very close to the theoretical value of 33%.

### 5.2. Comparison with the United States economy (1960-2000)

The chosen period is 1960-2000, for which we possess precise annual data<sup>35</sup> on growth of GDP and employment (in hours worked), and on the investment rate. The data used are presented in the appendix 2 (Table 3). We continue to consider the growth model with a profit share of 1/3<sup>36</sup>.

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assume that the system ever will get to an equilibrium; thus the fixed or moving equilibrium in the theory must be understood as an “attractor” rather than a characteristic of where the system is” (p. 66).

<sup>31</sup> On the short term, economic growth higher than maximal growth path can be obtained, as can recession; these extreme cases can be interpreted as situations when the capacity utilization ratio is temporarily rising or falling while the growth multiplier still lies between 0 and 1. Thus the equivalent growth multiplier would be higher than 1 or negative.

<sup>32</sup> Annually the share varies between 31% and 40%.

<sup>33</sup> This 34% average concerns a set of 41 countries, the profit share in the income varying from 20% to 35%.

<sup>34</sup> 1961-1970 : 30.2% ; 1971-1980 : 30% ; 1981-1990 : 31.3% ; 1991-2000 : 32.8%.

<sup>35</sup> The data sources are the World Bank (World Development Indicators-WDI) for the GDP growth rate and the gross investment rate, and the Groningen Growth and Development Centre (Total Economy Database, January 2007, <http://www.gdgc.net>) for the growth rate of the total number of hours worked. As the data bases do not provide net investment information, it has been presumed that the proportion of replacement investment is the classical 30%.

<sup>36</sup> The mean profit share in income (1960-2000) is 31.1% (European Commission, 2002).

Our knowledge of the average annual growth rates for GDP and employment, and of the investment rate, for the period 1960-2000, enables deduction of average values for the productivity of the investment capacity and growth multiplier (Table 2):

$$A = \frac{2}{i_n} \left[ \frac{\dot{Y}}{Y} - \frac{\dot{L}}{L} \right] \quad x = \frac{1}{Ai_n} \frac{\dot{Y}}{Y} \quad (31)$$

United States economy	1960-2000
<i>Fundamentals</i>	
Annual growth rate of GDP	3.41%
Annual growth rate of employment	1.64%
Investment rate (net)	0.131
<i>Growth model parameters</i>	
Productivity of the capacity investment ( $A$ )	0.27
Growth multiplier ( $x$ )	0.96

Table 2 - The fundamentals of the United States economy (1960-2000) and the growth model

It turns out that the average fundamentals of the American economy are close to the maximal growth path, characterized by a growth multiplier of almost 1.

According to the theory, the steady states (attractor) must verify the fundamental output-employment relationship:

$$\frac{\dot{Y}}{Y} = \frac{\dot{L}}{L} + 0.0177 \quad \frac{\dot{Y}}{Y} = \frac{\dot{L}}{L} + 0.135i_n \quad (32)$$

#### The fundamental output-employment relationship (by linear regression)

I shall verify the existence of such a correlation between, on the one hand, the annual GDP growth rates and on the other the annual employment growth rate (in hours worked) and the net investment rate. The correlation between output and employment is significant,<sup>37</sup> as is that between output, employment and net investment:<sup>38</sup>

$$\frac{\dot{Y}}{Y} = 0.94 \frac{\dot{L}}{L} + 0.0188 \quad \frac{\dot{Y}}{Y} = 0.92 \frac{\dot{L}}{L} + 0.145i_n \quad (0.10) \quad (0.0025) \quad (0.11) \quad (0.020) \quad (33)$$

Annual GDP growth rates correlate well with the employment growth rates and with the net investment rate, the employment coefficient remaining very close to 1.<sup>39</sup> Examination of the annual performances of the American economy for the period 1960-2000 confirm that employment growth rate and the investment rate correlate closely with GDP growth rate.

It appears that relations (32) and (33) are very similar. Figure 5 shows the annual positions on the economic trajectory for the period 1960–2000, together with the fundamental output-employment relationships. These latter were obtained theoretically from average annual values over the period (relation 32), and empirically by linear regression (relation 33).

<sup>37</sup> The values in parentheses are the standard errors for the coefficients. The R2 is 0.68. The statistics of Student T are respectively 9.00 and 7.48.

<sup>38</sup> The correlation appears significant given the standard errors (in parentheses) for the coefficients, and the statistics of Student T, respectively 8.61 and 7.34.

<sup>39</sup> Interestingly, Bernanke and Parkinson (1991), in their study of changes in production and employment in ten industries for the periods 1924-1939 and 1955-1988, found in the linear regressions an average employment coefficient of 1.07 and 0.96 respectively. In 72% of cases the coefficients for the different sectors fall between 0.8 and 1.3. They are obtained from quarterly observations for each of the ten industries.

Figure 5 illustrates the unbalanced character of annual economic growth and the role played by the steady states. The trajectory of the fundamentals curls around steady states which then appear to play the part of an attractor; the long-term average values of the fundamentals are consistent with those of the steady states. This reflects the fact that the instabilities are, in a way, channeled around the long-term relationship characterizing the stationary states of the growth model.

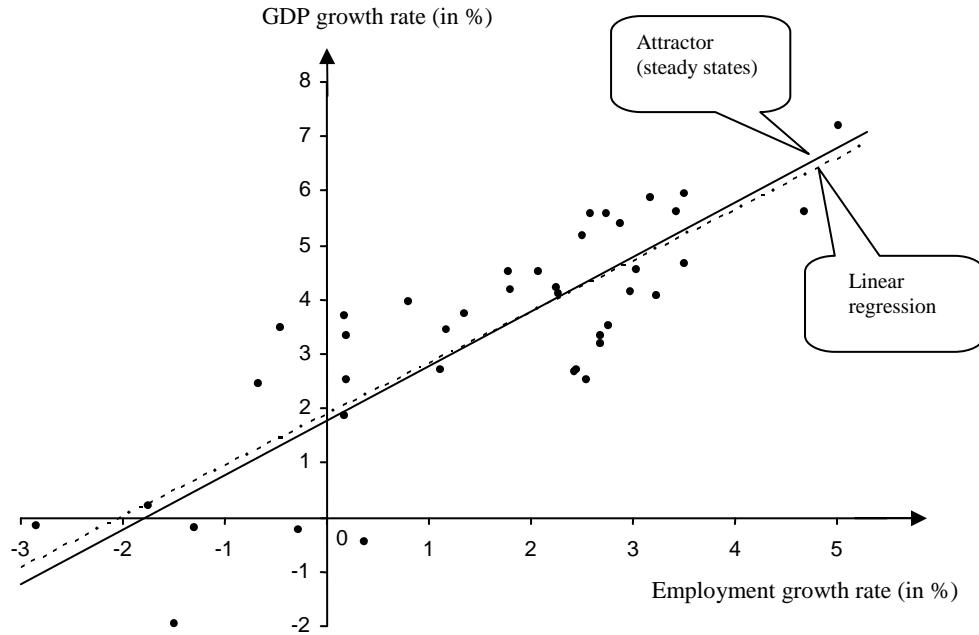


Figure 5 - The United States economy (1960-2000) and the steady states

## Conclusion

Kaldor's view of a chain-reaction and the complementary points of view of Schumpeter and Keynes have been represented by a new entrepreneurial growth model based on the principle of effective demand. It has been shown that, in the long term, this process admits steady states with unexpected properties.

In the steady states, output and employment growth rates verify the following fundamental output-employment relationship:

$$\frac{\dot{Y}}{Y} = \frac{1-c}{2c} \frac{\dot{L}}{L} + \frac{A}{2} i_n \quad \text{avec} \quad i_n = s_n = \text{constant}$$

in which  $A$  is the productivity of the capacity investment,  $i_n$  the net investment rate (or net saving rate) and  $c$  the profit share in the income. It has been demonstrated that the steady states are theoretically characterized by a profit share in the income equal to  $1/3$ , the independence between wage growth and employment growth being assumed. The trajectory of the growth paths curls around the steady states which then appear to play the part of an attractor for the long term.

The theoretical lessons to be drawn from the model prove consistent with the stylized facts provided by Kaldor and Barro and Sala-i-Martin, and with the stylized fact of a constant value - around  $1/3$  - for the profit share in the income. For the period 1960-2000 the American

economy is characterized by a fundamental output-employment relationship that matches the growth model.

Eventually, those results reinforce the Schumpeter, Keynes and Kaldor's complementary views for understanding economic growth.

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## APPENDIX 1- The competitive productive combination

### ***The lower cost***

In the context of capacity investment, the entrepreneur chooses the productive combination which minimizes the anticipated total cost per unit of additional supply. The anticipated total cost ( $Cost^a$ ) includes the cost of the increase in employment - the wage  $\omega$  being assumed constant - and the cost of the capacity investment, given the required marginal efficiency of the capital  $e_K$ . The anticipated total cost per unit of additional supply is:

$$\frac{Cost^a}{Ax^a I_n^a} = \frac{\omega L_c^a + e_K x^a I_n^a}{Ax^a I_n^a} = \frac{(1-c)}{A} \varepsilon_c^a + \frac{1}{A} e_K \quad x^a \neq 0 \quad (\text{A1.1})$$

$c$  being the profit share in the income, so that  $\omega L = (1-c)Y$ . The entrepreneur's aim is then to determine the optimal productive combination  $(\varepsilon_c^a, e_K)$ . The constraint indicates that the anticipated cost of the creation of employment, per unit of capacity investment, is inversely proportional to the marginal efficiency of capital:

$$\frac{\omega L_c^a}{x^a I_n^a} = \frac{C_1}{e_K} \Rightarrow \frac{\omega L \varepsilon_c^a x^a I_n^a}{Y x^a I_n^a} = (1-c) \varepsilon_c^a = \frac{C_1}{e_K} \Rightarrow \varepsilon_c^a e_K = \frac{C_1}{(1-c)} = C'_1 \quad (\text{A1.2})$$

The minimization program is thus equivalent to:

$$\text{Min}\{(1-c)\varepsilon_c^a + e_K\} \quad \text{subject to} \quad C'_1 = \varepsilon_c^a e_K \quad (\text{A1.3})$$

The solution is readily obtained by substitution of the constraint in the function to be minimized:

$$f(x^a, e_K) = (1-c)\varepsilon_c^a + e_K = (1-c)\frac{C'_1}{e_K} + e_K \quad \frac{\partial f}{\partial e_K} = -(1-c)\frac{C'_1}{(e_K)^2} + 1 = 0 \quad \frac{\partial^2 f}{\partial(e_K)^2} > 0 \quad (\text{A1.4})$$

The minimum<sup>40</sup> is such that:

$$\varepsilon_c^a = \frac{e_K}{1-c} \quad e_K \leq (1-c)\varepsilon_c^{mx} \quad (\text{A1.5})$$

### ***The expected profit***

Taking the optimal productive combination  $(\varepsilon_c^a, e_K)$  into account, the entrepreneur aims to determine the growth multiplier so that the anticipated profit (for all the investment) is equal to the required profit  $e_K I_n^a$ :

$$c(Ax^a I_n^a) = e_K I_n^a \Rightarrow x^a = \frac{e_K}{cA} \quad (\text{A1.6})$$

<sup>40</sup> The solution can also be obtained by writing as equal between the two terms of the sum to be minimized.

The entrepreneur admits that the profit share in the income is equal to  $c$ ; in other words, the anticipated wage growth rate is equal to the anticipated labor productivity growth rate.<sup>41</sup>

Two conditions appear, the first related to the maximum coefficient of employment creation and the second to the profit share in the income:

$$x^a = 1 \quad \varepsilon_c^a = \varepsilon_c^{mx} \Rightarrow \varepsilon_c^{mx} = \frac{cA}{1-c} \quad \varepsilon_c^{mx} < A \Rightarrow c < \frac{1}{2} \quad (\text{A1.7})$$

In the final analysis the competitive productive combination is characterized by the following relationships:  $x^a = \frac{e_K}{cA}$        $\varepsilon_c^a = \frac{e_K}{1-c}$        $\varepsilon_c^{mx} = \frac{cA}{1-c}$        $0 < x^a \leq 1$        $c < \frac{1}{2}$       (A1.8)

## APPENDIX 2 - Data used

1961	1962	1963	1964	1965	1966	1967	1968	1969	1970	1971	1972	1973	1974	1975	1976	1977	1978	1979	1980
2.47	5.17	3.97	5.58	5.60	5.95	2.72	4.19	2.67	0.22	3.46	5.59	5.88	-0.47	-0.18	5.38	4.66	5.62	3.18	-0.24
-0.66	2.53	0.80	2.60	3.44	3.52	1.13	1.82	2.43	-1.75	-0.44	2.75	3.18	0.37	-2.84	2.88	3.51	4.70	2.69	-0.27
17.5	17.7	18.1	18.6	19.1	18.8	18.1	18.3	18.5	17.9	18.4	19.2	19.6	19.0	17.8	18.2	19.5	20.8	21.4	20.4
1981	1982	1983	1984	1985	1986	1987	1988	1989	1990	1991	1992	1993	1994	1995	1996	1997	1998	1999	2000
2.52	-1.97	4.52	7.20	4.10	3.43	3.34	4.12	3.53	1.86	-0.19	3.34	2.69	4.06	2.54	3.75	4.55	4.22	4.49	3.69
0.19	-1.48	1.79	5.03	2.29	1.18	2.70	2.99	2.76	0.17	-1.30	0.20	2.46	3.25	2.56	1.36	3.04	2.27	2.08	0.19
20.1	19.0	18.7	19.7	19.4	18.8	18.5	18.2	17.4	16.3	16.2	16.7	17.2	17.7	18.2	18.6	19.1	19.6	19.9	

Table 3 - GDP growth rate, employment growth rate and gross investment rate (%) for the United States economy (1960–2000)

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<sup>41</sup> It is the difference between the anticipated supply growth rate and the anticipated employment growth rate.